Market equilibration, local vs global pricing measures and the onset of metastability

Abstract

We set up a dictionary that translates within a triptych of assumptions about market equilibration speed, the permanence properties of absence of arbitrage under formation of portfolios of compound assets and a hierarchy of obstructions to feasible computation in calibration tasks. We find that in a structural modeling approach to financial re-intermediation, arbitrage opportunities are generic instead of special, both from a bottom-up perspective and from a coherent view on stylized facts of long-range correlations in financial data. We discuss requirements for a new generation of market models that accomodate these facts and their compatibility with accounting, auditing, regulatory and corporate governance demands. We find that both the choice of business models and the implementation of oversight functions require the statistics of critical phenomena (or one of its avatars) in order to make possible the avoidance of gridlock in these tasks at a level of confidence compatible with the self-consistent structure of the financial intermediation process.

JEL Classification: A12, C18, C52, D82, D84, G21, G28

1 Introduction

We provide a bottom-up analysis of why market equilibration is trivialized in the arbitrage free contingent claim pricing paradigm and why this negligence is far from harmless but to the contrary, a major source for financial instability in the form of looming debt crises and contagion risks, stemming from the entangled web of debt contracts. We also show in the process that naive approaches to regulation fare no better. This conclusion is then reconsidered from the point of view of stylized facts on financial data and is found to match the econophysics explanations of those stylized facts, but not the view prevailing in the mathematical finance literature.

This clearly seems to leave us between a rock and a hard place. But not all is lost because *there is* indeed an extension of the prevailing paradigm that can at least potentially meet the challenge. Building contingent claim pricing and asset pricing models in the presence of arbitrage is extemely difficult, and will no doubt cut down the complexity of banking activity considerably. This however need not be detrimental to growth or even to the size of the financial sector at all, once the full rationale for its implementation is understood - together with its inevitability.

The main contribution of this paper consists in putting together the implications of the individual works referenced in the line of argument spanning a highly interdisciplinary problem. For this reason we omit a literature review and provide a refined table of contents instead of a walk-through of the argument in this introduction.

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2 Part I: Theory.

2.1 Toy model: arbitrage from violation of the principle of adequate variety

The translation of the high-level risk management *principle of adequate variety* into the calibration process of a market model in the framework of martingale-based contingent claim pricing states that in order to ensure consistency we need to match the number of driving random factors modeled to the dimensionality of the system of prices generated by the calibration instruments. If we use too few driving factors, we will obtain arbitrage opportunities within our model.

This meta-principle of implementation of equivalent martingale measures directly leads to reasons why financial re-intermediation often produces arbitrage: the funding instruments of large financial institutions are rarely replicated as contingent claims written against the full asset side dynamics, but are mostly represented within intensity based models for bank debt, following constraints only in the debt and CDS market, possibly linked to equity but usually not in a fully structural manner - which is most often practically impossible due to the large dispersion of creditors and maturities among funding instruments.

One possible reason for an insufficient number of factors would be a naive understanding of the regulatory framework - which itself suffers from a brute-force reduction of complexity down to the asymptotic single risk factor model. The reason for that reduction in the regulatory framework is Gordy's theorem saying that well-defined and portfolio independent capital charges require the single factor assumption. But this is a constraint coming from feasibility of the implementation of regulation, not a constraint on market dynamics.

These two potential sources of arbitrage are rather simple, of course. In a population of profit-seeking agents, it is likely that their price impact is transient and will dissipate quickly, to the point of being negligible most of the time. In other words, while these cases may very well exist (and at the smallest scale of resolution they even *have to* exist), it is plausible that they are detected quickly and 'integrate out' upon passage to larger scales of observation, or implementing a cut-off on frequency and resolution of market micro-structure parameters like time-scales and price-ticks.

But there are limits as to how far this dissipation can go, as we shall see now.

2.2 A bitter pill to swallow: hardness of internal treasury tasks and market equilibration is intrinsic

If we zoom in on the modeling tasks that need to be accomplished in order to profit from mispricings, we find a slightly less sanguine picture, a theme going back at least to Shleifer and Vishny (1997) and Grossman and Stiglitz (1980). We now update these findings and strengthen the skepticism considerably in the process.

2.2.1 Characterizations of the asymptotic single risk factor model and its extensions describing contagion

The framework for the Basel III credit portfolio model underlying *Pillar 1* is basically unchanged from Basel II and consists of the asymptotic single risk factor model, going back to Vasicek (2002). An enhancement of the model which includes the possibility to model contagion was given in Neu and Kuehn (2004) and is recommended as a feasible approach to measure an appropriate add-on reflecting contagion risk in the context of *Pillar 2*. In particular, the model by Neu and Kuehn allows for arbitrary deviations from granularity and uniformity assumptions and hence also exhibits the ability to produce highly multimodal loss distributions and phase transitions which would be typical as soon as one would allow multiple factors instead of one.

But the phenomenology of the model by Neu and Kuehn is already extremely rich both from a computational point of view and from the point of view of realistic descriptions of the actual task that internal treasury faces on an ongoing basis:

- as stated by the authors, the model belongs to the family of spin glasses and hence has the property that closed form solution or even numerical simulation at arbitrary high precision are *hard* problems, and by the results from Mezard and Montanari (2009) we may speak of NP completeness in the sense of computational complexity theory - unless special assumptions are warranted.

- if we consider the problem of building up a synthetic portfolio of exposures towards a given portfolio of names and consider the process of pricing tranches of debt written against that pool, dynamically managing maturing contracts on the asset and liability side, replacing them by new ones of comparable risk profiles, then we have described a stylized version of what internal treasuries do - at the same time we described an instance of the model.

The conclusion is that the generic situations that internal treasuries find themselves in are hard[1] already computationally - i.e. abstracting from the uncertainty of the origination and management process, unless special choices are made.

A close cousin of this argument is that the checking for absence of inconsistencies in a given specification of the full dependence structure of a general portfolio of names from a market of contingent claims written against that pool is a problem that can be mapped to the satisfiability problem (after forgetting a lot of the detail, i.e. it leaves an instance of the satisfiability problem after applying some forgetfulness) and is hence not tractable either, unless special assumptions happen to apply.

2.2.2 Models for contingent claims based on Levy processes vs Ito processes - a challenge for structural models

Intensity models for defaultable debt represent the latter in general as Levy processes that are not Ito processes, due to the presence of jumps. If we view the debt as in the Merton model as a short put option on assets, then debt generally follows an Ito process if the underlying (i.e. the asset side) does. With the possible exception of the time close to maturity, the process is hence much gentler than the general case of an intensity model.

More generally, if we have instruments modeled by general stochastic volatility models, such as those introduced by Barndorff-Nielsen and Shephard (1999) for example, then it may be difficult to distinguish these dynamics from an al-

[1] We really only care that these problems are NP, not whether they are NP complete or NP hard. In the sequel we therefore speak loosely about hardness with this understood.

ternative description in terms of an option on a high-dimensional (and possibly compound) underlying following an Ito process.

This difficulty exists already in case we assume that both modeling alternatives are compatible with the existence of an equivalent martingale measure. Once we drop that assumption, we find that markets 'built' from Ito processes and their compound contingent claims may exhibit arbitrage opportunities and may result in market dynamics exhibiting predictable jumps resulting from the arbitrage, while an alternative model based on Levy processes may 'view' the entire scenario as compatible with a richer Levy process environment without arbitrage.

We will look at a more involved construction of a similar kind in the next section. At this point, we do not claim that the Levy modeling approach could not detect the arbitrage opportunity. We merely point out that fitting a rich and complex market dynamics may also be the (inefficient) outcome of side-stepping the question whether the assumption of absence of arbitrage is warranted in the first place.

2.2.3 Hardness and NP completeness results on finding Nash equilibria in games

We may view a market that is free of arbitrage as a stage towards a complete market, where the completion expresses aggregate preferences of market participants, jointly driving the required processes of market price of risk.

In that view, the problem of reverse-engineering the (in general multi-dimensional) process followed by the market prices of risk is a translation of the problem of jointly optimizing the preferences of the market participants.

In case we have no assumptions about the consistency of these preferences, there is no reason why this process should be tractable, and in the worst cases (which can be constructed abstractly) it is not.

This perspective is a cousin of the intractability of the problem of checking consistency of an unconstrained dependence structure for a portfolio of obligors in the extension of the ASRF model containing contagion, and the two can be mapped onto each other under suitable background hypotheses.

2.2.4 Statistical mechanics of minority games and heterogeneous agents

In the wake of the original idea from Lux and Marchesi (1999), an industry of econophysics models has thrived which explain the microstructure of market equilibration from the El Farol bar Problem and the statistical mechanics of minority games - a vast generalization. In this class of models, the influence of market ecology is explicitly studied, including the impact of assumptions about diverse levels of information and cognitive depth across agents. These models produce bottom up explanations for the stylized facts of financial time series, but have often been criticized on the grounds that they do not fit into the paradigm of arbitrage-free pricing, or at least not obviously so - sometimes they don't by the very assumptions. The inverse conclusion that the high performing fit obtained with these models would imply arbitrage seemed barred. Even more so since the class of models obtained by Barndorff-Nielsen and Shephard (1999) and Comte and Renault (1998) could achieve a long memory of volatility while allowing the existence of equivalent martingales.

It seemed that this discussion had run into a stale-mate. While the most convincing models for market microstructure had much room for arbitrage at the smallest scale, mathematical finance insisted that market models that do not admit for equivalent martingales are suspect (or useless) at worst, and probably hard or impossible to calibrate at least.

This state of the discussion has changed somewhat with the arrival of the challenge posed by Albanese et al (2011), to the effect that the real question is whether equivalent martingales exist for the whole asset universe to be treated on the same footing in a useful internal treasury model which can also handle countparty risk, and hence must to some extent also model balance sheet dynamics of counterparties, at least on a coarse-grained level. We discuss this in the next subsection separately.

2.2.5 Local vs global martingale measures and the proliferation of financial instruments

Building upon Albanese et al (2011), Bardoscia, Livan and Marsili (2012) show that in a large market with many agents employing merely local martingale measures and a large number of financial instruments, the market dynamics itself will have as its likeliest outcome a state with global arbitrage opportunities.

While this appears to us to be the strongest and most universally applicable result challenging the paradigm of arbitrage-free pricing, we will add another construction in the next section to convince readers that the probabilistic nature of the result from Bardoscia et al (2012) by no means implies that the arbitrage opportunities are somewhat esoteric or merely due to some of the often disliked foundational assumptions of statistical mechanics.

In the process we will also explicitly resolve the apparent paradox in view of the existence the models by Barndorff-Nielsen and Shephard (1999) and Comte and Renault (1998).

2.2.6 Operational risk modeling

While phase transitions are to be viewed as a worst case outcome in contagion modeling for credit portfolio risk, power laws of the generalized Pareto family are seen as a given when it comes to finding best fits for operational risk. This means that not only fat tails are a necessity for at least some of the funding instruments of banks (unless operational risk could be fully hedged or insured against - which is not the case), but also that tails with divergent moments are. Hence, in a sense, operational risk provides another source of the kind of phenomenology which has been associated with hard-to-detect arbitrage opportunities and obstructions against the existence of equivalent martingale measures.

There's another twist why operational risks are close to the spirit of the present paper: the relation between the core problem of computational complexity theory, namely the challenge whether $NP \neq P$ and its relation to the foundations of failure of e-commerce and e-banking. In a sense, banks either have an operational risk they can't possibly handle or a business risk they can't possibly handle. One may reasonably ask the question: who is footing that bill? Is it priced? If so, how? Based on what model? In fact it seems that there's a downright reductio ad absurdum hidden here. Either banks will one day face a breakdown of the business model of e-banking faster than they can scale down their business, or they will forever remain unable to find tractable routines to test for the soundness of the implementations of their IT infrastructure. The latter is by far the more likely case. But why are we supposed to trust that they are infinitely strong arbitrageurs then? This provides a very strong case not only for the separation of lines of business, but beyond that, for much stronger relations between arbitrage activity within banks and the appropriate oversight to which they must be subjected.

2.2.7 Computational complexity and the sorcerer's apprentice

Arora, Barak, Brunnermeier and Ge (2009) show that computational complexity can also serve as a tool in the design of financial products that carry very strong potential for informational asymmetry and seem like a recipe to create lemon markets. Basically, structuring products without offering full see-through documentations is quite generally a recipe for disaster coming from computational complexity. Unfortunately, the same applies to the intransparency in the general banking sector even without synthetic structures, simply because the web of interconnected indebtedness produces a natural example for how to realize structured products. From that perspective, it is not very surprising to find that interbank markets have had difficulties ever since the financial crisis.

2.3 A coherent view on risky assets, defaultable debt and AAA securities: generic arbitrage from long-range correlations

2.3.1 Long-range correlations in the Lux-Marchesi model

To anchor our argument in empirical data, in addition to the theoretical reasons we offered, we could take the most convincing explanation of the stylized fact of long-range correlations in financial time series as a starting point - the class of models building on the original idea of Lux and Marchesi (1999) - and show that under suitable (but weak) extra conditions it is already implied by the facts, not merely vice versa. Since that explanation involves a persistent asymmetry of information and an information-driven cascade of price-movements, we will then automatically have a situation where the behaviour of different segments of the market is described by measures corresponding to filtrations representing distinct degrees of information, which in the contingent claim pricing literature is known to imply arbitrage, being an example of insider trading. But in order to avoid any impression that the facts are in any sense related to illegal actions and in order to remove all semblance that the phenomenon admits for alternative interpretations involving only divergent preferences (instead of divergent information), we will choose a different approach.

Interestingly, several authors have taken the opposite direction and have responded to criticisms of long-range correlations in models admitting for arbitrage by modifying the models such as to fit into an equivalent martingale framework. This sequence of events seems rather odd, since we must suspect that the best way to remain sure that mispricings die out is to have a class of models at hand that can 'remove' them not by assuming them away but by bidding them away, with a definitive price impact.

2.3.2 Long-range correlations in the Barndorff-Nielsen Shepherd (BNS) model

We take the rationale of information cascades in the class of models introduced by Barndorff-Nielsen and Shepherd (1999) - which were embedded into an arbitrage-free modeling context based on Levy processes from the beginning - and will provide a fairly general argument for the following

Surmise 2.1 *Assume that an asset universe contains*

- *risky assets taken from the BNS class, in particular exhibiting long memory of volatility*
- *riskless assets represented by a term structure model*
- *sufficiently many contingent claims written on the underlying BNS assets to hedge against all fine detail inherent in their volatility processes (i.e. a completion which abstractly exists under the assumption of absence of arbitrage)*
- *a completion of all term structure risks.*

We assume furthermore that the asset universe is free of overall arbitrage.

Then a portfolio fully hedged within this asset universe against all sources of market price of risk (using up the resources of the contingent claims assumed to be correctly priced in the completion process) essentially evolves like a classically (deterministic) chaotic system.

The surmise generalizes the known fact[2] that in the stochastic volatility models by Heston (1993) and by Comte and Renault (1998), the degeneration in the low volatility regime fails to be 'graceful', i.e. there are constraints on the parameter values beyond which volatility cannot drop without leaving the range where absence of arbitrage can be imposed without contradictions.

[2] cp. Henry-Labordere (2009)

Maybe one could also view the numerical instability of the calculation of higher moments in the low volatility regime in the Standard Black Scholes model as a glimpse of this phenomenology.

2.3.3 The rationale for the surmise from physics

In quantum mechanics, the problem is the known difficulty of graceful passage to the classical limit. In quantum field theory, especially quantum chromodynamics (QCD), or in the statistical field theory of the Ising model or spin glasses, the analogous problem is the computational complexity of determining confinement (or its absence) in the infrared - a phenomenon which is itself the source for the versatility of these models for purposes of universal computation.

Generally speaking, the behaviour expresses the fact that the passage to the classical limit is a specialization or a degeneration, while quantization is a smoothening: the complexity of the time-correlation structure of classical limits doesn't drop upon setting the Planck parameter[3] $h \mapsto 0$ - if anything, it typically increases.

In the QFT case the increase of complexity in the infrared is even more pronounced. It leads into the subject of quantum criticality and the study of long-range correlations via the gauge-gravity duality and the holographic principle, cp. Klebanov and Maldacena (2009). In this context the proliferation of metastable states, i.e. the complexity of the fine structure of the vacuum is known to be at the heart of many unsolved problems, including their reflections in mathematical physics, which include the conformal field theory approach to statistical mechanics, i.e. the Langlands program.

The hardness of the task of determining the structure of the vacuum in the presence of long-range correlations then translates into the much-cherished

[3] which of course is a constant in physics. Never mind. In Baaquie's (2004) quantum finance, the volatility v is expressed as h/m where m is a parameter corresponding to mass in physics. So another way for obtaining a low or vanishing volatility regime is for the mass to increase beyond limits - unless h changes as well...

wealth of content of Trace Formulas.

2.3.4 The economic rationale for imposing the requirement of 'graceful degeneration'

The economic rationale for demanding the gracefulness of the degeneration is as follows.

First of all, from a modeling building perspective, it is hard to even argue about the requirement, since in the absence of arbitrage, completion is naturally the next step and is also needed in order to make full use of the model. The point is that the process of hedging gradually against all traded risks and their market price of risk is a toy descripton of the internal treasury function. Even if a directional bet remains, the process must be possible for internal treasury to be able to fulfil its function of arbiter of internal transfer prices and breakdown of funding costs to business lines across the spectrum of maturities.

Furthermore, re-intermediation naturally produces a new class of assets - deposits and senior tranches of debt - which by the very definition of target levels of solvency in bank regulation are candidate securities for the benchmark rate of near-riskless AA debt across the maturities. If the CDS market is furthermore assumed to be arbitragefree itself and to fit into an overall arbitrage-free and structurally well-grounded model, then the degeneration can be practically implemented in markets and must hence be consistent.

Still another reason - close in spirit to the original BNS model - is that upon rising of asset prices, we have another stylized fact, the leverage effect, so that a low volatility regime is naturally part of the full asset dynamics. To wit, one may even speculate about an intrinsic relation between the limits of the consistency of the modeling approach that we found and the stylized fact of increased volatility as asset prices start to drop beyond their 'calm' summit. In other words: it is not implausible that our observation can be jazzed up towards a model for the microstructure of the bursting of asset bubbles.

2.4 Universality classes of market equilibration speeds - a qualitative approach

2.4.1 Universality classes of critical phenomena

One of our main conclusions in this paper will be that both the choice of business models and the implementation of oversight functions require the statistics of critical phenomena or one of its avatars. We conclude this from a characterization of equilibration speeds of markets in qualitative terms, and establishing various terminologies for the stratification of the set of possible states of markets. This approach is inspired by an interdisciplinary paradigm that permeates various fields of study, and which we mention here, before referring the reader to the vast literature on the subject. It will be clear in a moment why a full guide to the literature would not fit into the format of this paper - nor is the author qualified to give one. That may seem paradoxical, but it will also be clear in a moment why it's not, the short answer being that there is no alternative to the difficult task of staying afloat in these waters - which is precisely what we are saying in this paper.

Since the difficulty is an Escherian one, we may as well start in medias res, at an essentially arbitrary point of entry. One such point of departure is the russian doll character of financial intermediation: securities funding banks are contingent claims written against an underlying that itself contains debt and equity (and of course other derivatives), hence contingent claims written against *another* underlying, to be analyzed - or not, once we opt for a cut-off in our pricing methodology. Hence we find the signatures of complex systems, self-organization, fractals, or renormalization group flows and their fixed points in the field of financial intermediation. In particular this provides a general-purpose method to explain the ubiquity of power laws in finance, and explains the success and empirical adequacy of extreme value theory and econophysics methods, as well as the failure of the purely gaussian paradigm.

But unfortunately, taking this point of view has the inherent tendency to open up still more black boxes in the process.[4] In other words, a 'unified' view on these phenomena eludes us, and has done so for every step along the way during the past half century, in which it has been rediscovered (and re-'unified') roughly ten times per decade, across economics, finance, physics, mathematics, computer science, biology, cognitive science, logic, epistemology and philosophy. In a non-trivial sense, it could be said that from the point of view of the history of science, the best possible explanation for the persistency of the efficient market hypothesis, the gaussian paradigm and the insistence on assumptions of absence of arbitrage in contingent claim pricing is the very hardness of formulating a coherent alternative paradigm. What we do know, however, is that this hardness is a perfectly well-established fact. Since not even the task of providing an overview is tractable we refer to the literature at this point, for example Haken (1990), Lesne (1998), Sornette (2006).

The challenge posed by this hardness or intractability for science and interdisciplinary efforts is harmless of course. To the contrary, it justifies these efforts and will continue to do so, independently of the properties of market equilibration. As we shall see now, the case of market equilibration is special, however.

2.4.2 The case of markets

What's special about the case of market equilibration speeds as an example within the universality classes of critical phenomena is that the rationale for the view that markets are essentially efficient and free of arbitrage is *incompatible* with the hardness and intractabilities that we found to be persistent. Because that rationale is the view that the profit-seeking motive of sufficiently many

[4]some of which behave statistically like, to wit, black holes. This is the content of the BHQC, the black hole qubit correspondence of Borsten, Duff and Levay (2012). This is closer to our subject than it seems. For example, the computational complexity of spin glass models is a close cousin of the computational complexity of the landscape, as is shown in Denef and Douglas (2006).

sophisticated economic agents suffices to establish that arbitrage opportunities are self-extinguishing. The limitations of the rationale would still leave open the possibility that markets *happen to be* essentially efficient and free of arbitrage as a mere matter of fact, most of the time, without the rationale. But the closer look at the stylized facts of long-range correlations has revealed that such a point of view (which would a priori be a kind of last intellectual resort) is fragile as well: the kinds of asset universes in which this point of view is valid are arcane. While it may be true that it is in the very nature of markets of risky assets that their volatility exhibits long-range correlations due to the self-organized nature of the intermediation process, this fact itself is best explained as the trace of the ongoing removal of arbitrage opportunities rather than by their absence. And if we take this point of view to be basic, then we also face the fact that the glueing of risky asset markets and riskless (or fully hedged) investment portfolios is bound to be burdened with frictions of its own, since the 'graceful' degeneration is not to be had for free, and admits consistent descriptions only in rarefied conceptual environments.

This is, at bottom, why both the designers of business models in financial intermediation and their oversight bodies cannot do without the phenomenology of the statistics of critical phenomena. The only way out is further in.

A stratification of market phases that is minimal with respect to our purposes in this work comprises the following three

- arbitrage-free and completeable (informationally efficient, in equilibrium)
- self-organized, i.e. there exist well-defined arbitrage-free candidate nearby hypothetical markets (ideally a single one) such that the actual market can be meaningfully interpreted as a perturbation, equilibrating and possibly regime-switching between the candidates
- (the stochastic analog of) chaos, i.e. no meaningful long-term candidate arbitrage-free market exists, not even a hypothetical one, for example due

to an infinity or continuum of solutions to the number of regimes in regime-switching frameworks

Most important for our application are the boundaries separating the middle (or generic) one from its two specializations:

- The self-organized phase with a single equilibrium candidate solution becomes equivalent to the first case for increasingly fast speeds of convergence.
- For an increasing number of equilibria and / or divergent times towards equilibration, the self-organized phase undergoes the onset of metastability and in the limit suffers the breakdown of the validity of the principle of maximization of information entropy.[5]

While in the first degeneration the assumption of absence of arbitrage remains a reasonable approximation, the unfortunate fact is that the stylized fact of long-range correlations puts us mostly into the second boundary, i.e. into the 'edge-of-chaos'. Most of our applications naturally take up the question of how to steer clear of that limit, under constraints imposed by the real world of market microstructure as well as the real world of institutional designs and incentives of accountability and reporting.

Any one of the available terminologies for the statistics of critical phenomena comes with its own disadvantages (and some advantages of course). The synergetics terminology of 'self-organization' suffers from the double confusion that arises from the application of the principle of adiabatic elimination of fast variables (which is primarily an equilibrium principle) in out-of-equilibrium situations, explaining the emergence of order by a principle that asks us to maximize disorder. Resolutions of these terminological traps have been offered for example in Gaspard (2005) and Ovchinnikov (2012). The textbooks Mezard and Montanari (2009) and Mussardo (2010) offer robust terminologies for discussing the

[5] or of the principle of adiabatic elimination of fast variables, in Haken's synergetics terminology.

same stratification as the onset of the proliferation of metastable states or the classification of critical phenomena in terms of their associated conformal field theories.

2.5 Situating the new paradigm of market models, recovering old models as degenerate limits

We find that these difficulties leave no other option but to pass to a new generation of market models which explicitly allow for the presence of arbitrage opportunities, as long as these decay fast enough. At the very least, internal treasuries must measure the performance of the methods to extract market price of risk from calibration instruments with increased rigor.

It is clear that the class of instruments that can be treated from within such models is highly restricted at this point. Farinelli (2009) offers some examples. The justification for the gargantuan effort needed to extend mathematical finance to this level is given by the argument in the present paper, to the effect that this effort is nothing less than necessary. There is no alternative. The only way out is further in.

3 Part II: Application.

We may summarize Part I as follows:

The internal complexity of market equilibration at the level of individual assets, at the level of aggregation of risks via the internal treasury function in financial intermediaries, and at the level of the market as a whole (including a see-through version of the funding instruments of intermediaries) is such that the profit-seeking behaviour of arbitrageurs does not suffice to steer clear of a breakdown of the price mechanism at the confidence level required by the ratings of financial institutions' debt (including deposits, 'rated' by the target confidence level of bank regulation) unless we have reason to assume that equilibration takes place on 'special inputs'[6] for the activity of arbitrageurs. Markets might be special due to particular choices of business models or due to constraints effectively imposed by means of oversight. The idea that the profit-seeking behaviour of investors by itself rules out the disintegration of markets at the required confidence level is untenable, as is the idea that a rigid formula-based capital charge could be implemented without difficulty in a self-consistent manner.

The only viable option is to face the fact that market equilibration creates a state of permanent friction for any feasible and operationalizable method of pricing and risk management. We now discuss the consequences for management and oversight functions.

3.1 Accounting, auditing and disclosure

We have seen that prices and risks are driven by the flow of information and its obstructions, including hard computational ones. In particular, risk disclosure is a key order parameter that governs the tractability of market equilibration. Pricing and hedging of instruments must be embedded in an extra loop, first settling the time-scales of equilibration required for all risk factors towards which any given instrument is sensitive. The practice of holding assets at fair values extracted directly from market prices or implied from market prices must be complemented by a routine that measures the difficulty of calibrating the market price of risk factors driving the relevant asset, consistently across all exposures. There is no such thing as a fair value or implied valuation parameter that is reliably extracted from market dynamics without a test for the speed of equilibration of the entire market in which these market prices of risk are being traded. Auditors must have access to these tests and must be able to confirm their methodological soundness. The extra costs are internalized by requiring

[6] in the sense of an algorithm

auditors to report on the results of these tests, so that there is an incentive to reduce complexity of business models - or to increase their soundness.

In the absence of such a mechanism, self-sustaining bubbles are bound to happen, because the complexity of the profitable trading strategies 'containing' them is excessive already because there is no reason for the sustained mispricings to remain 'local'. To the contrary, they will typically involve mispricings of funding instruments of the financial sector as well, so that their detection is presently made harder already by the insufficient risk disclosure, which is too coarse-grained to allow for detection and elimination of all capital structure arbitrage opportunities in the interbank markets.

3.2 Ratings, regulation, central bank liquidity

Rating agencies must disclose proof how their past ratings can be brought in accordance with the assumption of absence of arbitrage in debt markets, at least at a coarse-grained level. If no such calibration is feasible, they must disclose how they intend to modify their modeling approach to accomodate those past scenarios. In that case they must also offer a see-through analysis of the web of entanglements of all entities that have issued debt rated by them, leading up until the present. In order to be able to do that, they must collect and gather information about the web of cross-indebtedness for all their clients. These clients in turn must therefore provide the relevant information to the rating agencies, which may include the full trace of past funding activity.

Central banks cannot assume that their mandate of securing target levels of inflation is independent of the absence of arbitrage in the market of funding instruments of the financial sector. The reason is that the role and definition of the riskless investment alternative is itself in question when bank funding instruments do not fit into arbitrage-free models of defaultable debt - which may simply elude depositors. Central banks must also monitor the speed of market equilibration on an ongoing basis in order to gauge the solvency of troubled banks instantly, whenever they offer or provide emergency liquidity.

In regulation, the internal capital adequacy process (Pillar 2) is key, for the following reasons:

- the mere self-consistent application of the regulatory capital charges is not even consistent with the absence of (capital structure) arbitrage under the assumption that ratings and LGD estimates do fit into a consistent market model

- diversification benefits for risk aggregation would be impossible under the assumption that the joint (single) market factor in the credit risk capital charges can be meaningfully interpreted as indeed representative of the aggregate market performance, because in that case the loss quantiles of the ASRF model are deterministic functions of the market risk quantiles, so that VaR would be additive in this case.

- the divergent moments of best fits to operational risk loss distributions signify intrinsic limits to the interpretation of these loss potentials as being 'earned' without imposing significant external effects

- any time somebody buys a solution to the NP=P problem at the price of one million dollars[7], e-banking breaks down

3.3 Corporate governance, stakeholder vs shareholder value

The shareholder-value oriented paradigm for corporate governance rests on the assumption that bondholders and outside parties can rely on the legal environment and the fulfilment of contracts alone in order to protect their interests. In the case of banks with their dispersed creditors, this paradigm is broadened to include the need for regulation or deposit insurance to protect the interests of creditors, cp. Dewatripont and Tirole (1994).

[7] offered for the solution of that millennium problem

What we have seen shows that not even that extended framework is sufficient, since the key to achieving a sustainable confidence level for the solvency of the banking sector as a whole, i.e. the key to providing the public good of a market in the self-organizing state is not in the hands of oversight entities alone. It requires a global constraint on the distribution of choices of business models of financial intermediaries as well.

It is a corollary of this choice of words that for a market to be self-organizing, it is far from enough for oversight agencies to just 'take their hands off the steering wheel'. But this terminological confusion has nothing to do with ideological preferences, of course. It results from the fact that the 'order from chaos' that self-organization can sometimes produce doesn't come automatically. There are conditions for it to arise, no matter how we call the phenomenon. And those conditions are *not always* satisfied. And unless oversight and the choice of business models satisfy some joint constraints, they are *not* satisfied. When they are not satisfied, the result is not a debt crisis that comes with certainty, but a situation in which the absence of crisis does not hold at the confidence level that is associated to the investment grade debt of banks. And such a situation refutes the reputational requirements of integrity that regulators, rating agencies, central banks, bank auditors and senior bank management must fulfil.

And yes, we *have been* in such situation. No amount of obfuscation in the ongoing regulatory reform process can remove that fact from the track record of the above-mentioned entities whose reputation is on the line. The process of gaining, maintaining, securing and regaining trustworthiness was not and is not a one-man show.

3.4 Business models and constraints imposed from external effects

The choice of business models is up to the owners or can be delegated to management if external effects are absent. So how are the costs for telling whether there are any external effects associated to a given choice of business model internalized?

The answer to this question would be redundant and the question would be purely academic only if there was a rationale why externalization of costs as a consequence of particular choices of business models wouldn't be potentially lucrative.

This is far from the case, however. We have seen that for financial intermediaries with an edge in computational prowess, the proliferation of financial instruments and the associated increase in market complexity, the resulting slow-down of market equilibration speed and the ensuing error-prone-ness of retail investor decisions or depositors are indeed lucrative.

Indeed, a profitable business model is to extract value from an inconsistency of the aggregate preferences of investors expressed in the dynamics of the market prices of risk. Another is the arbitrage opportunities to be extracted from the failure of graceful degeneration of markets of risky assets exhibiting long-range correlations when those are fit into the full picture which also includes fixed income markets and the riskless investment portfolio.

In other words, while banks may have an incentive to avoid full-fledged debt crises, they certainly don't have an incentive to make sure everybody understands finance well enough. So the costs for mitigating the lack of financial literacy are a possible externality of the choice of business models. The choice of business models is hence not solely up to owners or management.

When we increase the level of resolution, more externalities appear: not only towards the non-financial sector, but also towards remote parts within the sector, which may be affected by a meltdown in worse ways than a global player with an edge in taming complexity.

We conclude that the choice of business models is *not* a bilateral private decision fully protected by the freedom of contracts applicable between owners and managers - and disregarding other stakeholders, such as bondholders including depositors, governments and tax payers.

3.5 Systemic risk contributions and Living wills

Borrowing a phrase from Lo and Mueller (2010), we may now have reached the 'Zen-level' of Escherian ramifications of ill-posedness, hardness and intractability. But we need not be afraid. Because actuarial science has been there. And it tells us that we can't price or insure what we can't model or at least partially hedge.

The role of systemic risk contribution calculations and Living wills is largely symbolic. They tell us that unless we write with a pen that wouldn't get us into trouble even as we use it to paint new money, we have no business signing contracts that insure against systemic risk. Reification of risks is not a cure against the ills of proliferation of financial complexity. Monitoring the traffic lights system of market equilibration speeds at all times - including market calm - is.

Nevertheless we need to assemble the risk premia collected in accordance with the systemic risk contributions. Facing what we know about the complexity of the worst instances of market equilibration requires us to set up an institution securing the public good of an investor of last resort or representative rational agent with deep pockets - which is simply assumed for free in the old paradigm. This institution then also acts as a buyer of market risk (of last resort) as long as there is an abstractly well-defined market price for it, and is financed by the insurance premia in accordance with systemic risk contributions of players in the financial sector, but also endowed with access to financial leverage provided by the central bank. In other words, an honest and accountable banker, in case there is none.

Interestingly, this institution then also has an unintended consequence on the positive side: being a buyer of market risk of last resort, any bank that manages the process of risk transformation resonably can lend in all phases of the business cycle even under the higher regulatory capital constraints to be phased in under Basel III, simply by transforming credit risk into market risk (by replicating the debt contract of the loan in terms of a short put option on the asset side of the client) and then selling it to the investor of last resort if necessary, keeping only an idiosyncratic component that represents its informational edge but doesn't harm its overall capital position.

In the presence of such an institution, winding down failing banks becomes feasible. The Living will simply documents that the bank can explain to itself that it is in the business of transforming credit risk into market risk - because that's what a bank does, so it's a good thing to put this in a written form.

4 Conclusion

We found that the unconstrained problem of market equilibration contains a faithful representation of the worst case examples of the proliferation of metastable states, and is hence intractable - unless constrained or restricted to special instances. The public good of markets that remain within the bounds of deviations from arbitrage-free equilibrium amenable to tractable resolution is not free but comes at a cost. The process of providing this good even at a cost is non-trivial, since it requires reliable and dependable communication between those responsible for the choice of business models and those responsible for oversight.

References

[1] Albanese, C., Bellaj, T., Gimonet, G., Pietronero, G. (2011): Coherent global market simulations and securitization measures for counterparty credit risk, Quantitative Finance, Vol. 11, No. 1, January 2011, 1-20

[2] Arora, S., Barak, B., Brunnermeier, M., and Ge, R. (2009): Computational Complexity and Information Asymmetry in Financial Products. Working paper.

[3] Artzner, Ph., Delbaen, F., Eber, J.-M., Heath, D. (1999): Coherent measures of risk, Mathematical Finance, vol. 9, No. 3 (July 1999), 203-228

[4] Bardoscia, M., Livan, G., and Marsili, M. (2012): Financial instability from local market measures, arXiv:1207.0356 [q-fin.TR]

[5] Barndorff-Nielsen, O.E., and Shephard, N. (2001): Non-Gaussian Ornstein-Uhlenbeck-based models and some of their uses in financial economics, Royal Statistical Society B (2001) 63, Part 2, 167-241

[6] Baaquie, B.E. (2004): Quantum finance - path integrals and Hamiltonians for options and interest rates, Cambridge University Press, Cambridge, UK

[7] Bernanke, B., and Gertler, M. (1990): Financial fragility and economic performance, Quarterly Journal of Economics 105 (1), 87-114

[8] Borsten, L., Duff, M.J. and Levay, P. (2012): The black hole / qubit correspondence: an up-to-date review, arXiv:1206.3166

[9] Brunnermeier, M., Crockett, A., Goodhart, Ch., Persaud, A., and Shin, H. (2009): The fundamental principles of financial regulation, International center for monetary and banking studies, January 2009, Geneva, Switzerland

[10] Caccioli, F., Marsili, M., Vivo, P. (2009): Eroding market stability by proliferation of financial instruments, arXiv:0910.0064 [q-fin.TR]

[11] Comte, F., and Renault, E. (1998): Long memory in continous-time stochastic volatility models. Mathematical Finance 8, 291-323

[12] Denef, F., and Douglas, M.R. (2006): Computational complexity of the landscape I. arXiv: hep-th/0602072

[13] Dewatripont, M., and Tirole, J. (1994): The prudential regulation of banks. Cambridge, Massachusetts, MIT Press

[14] Duplantier, B. (2000): Conformally Invariant Fractals and Potential Theory, *Physical Review Letters*, vol. 84, Feb 2000, number 7, 1363-1367

[15] Farinelli, S. (2009): Geometric arbitrage theory and market dynamics, SSRN abstract 1113292

[16] Freixas, X., and Rochet, J. Ch. (2008): Microeconomics of banking - 2nd edition, Cambridge, Massachusetts, MIT Press

[17] Gaspard, P. (2005): Hamiltonian dynamics, nanosystems, and nonequlibrium statistical mechanics. Lecture notes for the International Summer School Fundamental Problems in Statistical Physics XI, arXiv: cond-mat/0603382 [cond-mat.stat-mech]

[18] Grossman, S., and Stiglitz, J. (1980): On the impossibility of informationally efficient markets, *American Economic Review*, vol. 70, no. 3, 393-408

[19] Greenspan, A. (1998): The role of capital in optimal banking supervision and regulation, *FRBNY Economic Policy Review*, October 1998

[20] Haken, H. (1990): Synergetik - Eine Einführung, 3. Auflage, Springer-Verlag, Berlin Heidelberg New York

[21] Henry-Labordere, P. (2009): Analysis, geometry, and modeling in finance, Chapman and Hall, Boca Raton, London, New York

[22] Heston,S. (1993): A Closed-Form Solution for Options with Stochastic Volatility with Applications to Bond and Currency Options, Review of financial studies 6, 327-343

[23] Jizba, P., Kleinert, H., and Haener, P. (2009): Perturbation expansion for option pricing with stochastic volatility, *Physica A*, 388, 3503-3520

[24] Klebanov, R., and Maldacena, J. (2009): Solving quantum field theories via curved spacetimes, *Physics Today*, American Institute of Physics, January 2009, 28-33

[25] Kitsukawa, K., Mori, S., Hisakado, M. (2006): Evaluation of tranches in securitization and long-range Ising model, arXiv: physics/0603040

[26] Krajewski, T., Martinetti, P. (2008) Wilsonian renormalization differential equations and Hopf algebras, Talk given at the conference 'Combinatorics and Physics', MPI Bonn, March 2007, arXiv:0806.4309

[27] Lando, D. (2004): Credit risk modeling: theory and applications, Princeton, Princeton University Press

[28] Lesne, A. (1998): Renormalization Methods - Critical Phenomena, Chaos, Fractal Structures. John Wiley and Sons, Chichester

[29] Lo, A. and Mueller, M.T. (2010): Warning: Physics envy may be hazardous to your wealth!

[30] Lux, T., Marchesi, M. (1999): Scaling and criticality in a stochastic multi-agent model of a financial market, Nature 397, 498-500

[31] Malevergne, Y., Sornette, D. (2006): Extreme Financial Risks - from dependence to risk management, Springer, Berlin and Heidelberg.

[32] McNeil, A.J., Frey, R., and Embrechts, P. (2005): Quantitative risk management, Princeton University Press, Princeton and Oxford

[33] Mezard, M., Parisi, G., Virasoro, M. (1987): Spin glass theory and beyond - an introduction to the replica method and its applications, World Scientific, Singapore New Jersey Hong Kong

[34] Mezard, M., and Montanari, A. (2009): Information, Physics, and Computation, Oxford University Press, Oxford

[35] Mussardo, G. (2010): Statistical field theory - An introduction to exactly solved models in statistical physics, Oxford University Press, Oxford

[36] Neu, P., Kuehn, R. (2004): Credit risk enhancement in a network of interdependent firms, *Physica A* 342, 639-655

[37] Ovchinnikov, I.V. (2012): Topological field theory of dynamical systems, Chaos 22, 033134

[38] Schoenbucher, P. (2003): Credit derivatives pricing models: models, pricing and implementation. Chichester, UK, Wiley

[39] Sornette, D., Andersen, J.A., and Simonetti, P. (2000): ϕ^q-Field theory for portfolio optimization: fat tails and non-linear correlations, *Physics Reports* 335, 19-92

[40] Sornette, D., Malevergne, Y. (2003): VaR-efficient portfolios for a class of super- and sub-exponentially decaying assets return distributions, arXiv: physics/0301009

[41] Sornette, D., Malevergne, Y. (1998): Multi-moments method for portfolio management: generalized capital asset pricing model in homogeneous and heterogeneous markets, arXiv: cond-mat/0207475

[42] Sornette, D., Zhou, W. X. (2005): Self-fulfilling Ising model of financial markets, arXiv: physics/0503230

[43] Sornette, D. (2006): Critical Phenomena in Natural Sciences - Chaos, Fractals, Selforganization and Disorder: Concepts and Tools, Springer Series in Synergetics, Springer, Berlin Heidelberg.

[44] Sornette, D., Holyst, J.A., and Sieczka, P. (2010): The Lehman Brothers Effect and Bankruptcy Cascades, arXiv:q-fin/1002

[45] Shleifer, A., and Vishny, R.W. (1997): The Limits of Arbitrage, The Jounral of Finance, Vol. LII, No. 1, March 1997, 33-55

[46] Vasicek, O. (2002): Loan portfolio value. *RISK magazine*, december 2002. 160-162

[47] Vilela Mendes, R., Oliveira, M.J., and Rodrigues, A.M. (2012): The fractional volatility model: No-arbitrage, leverage and completeness. arXiv:1205.2866 [q-fin.ST]

[48] Warner, N., Vafa, C. (1989): Catastrophes and the classification of conformal theories, *Physics Letters B*, Vol. 218, number 1, 51-58

[49] Zinn-Justin, J. (2007): Phase transitions and renormalization group, Oxford University Press, Oxford